LECTURE NOTES
ON
ELECTRICAL MEASUREMENTS
2018 – 2019
III B. Tech I Semester (JNTUA-R15)
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Course Objectives:

The objectives of the course are to make the student learn about

- The basic principles of different types of electrical instruments for the Measurement of voltage, current, power factor, power and energy.
- The measurement of R, L, and C parameters using bridge circuits.
- The principles of magnetic measurements.
- The principle of working of CRO and its applications.
- The use of Current Transformers, Potential Transformers, and Potentiometers.

UNIT- I

MEASURING INSTRUMENTS


UNIT – II

D.C & A.C BRIDGES


UNIT – III

MEASUREMENT OF POWER AND ENERGY

UNIT – IV

INSTRUMENT TRANSFORMERS AND POTENTIOMETERS


UNIT – V

MAGNETIC MEASUREMENTS


OUTCOMES: The student should have learnt how to
- Use wattmeters, pf meters, and energy meters in a given circuit.
- Extend the range of ammeters and voltmeters
- Measure active power, reactive power, power factor, and energy in both 1-phase and 3-phase circuits
- Determine the resistance values of various ranges, L and C values using appropriate bridges.
- Analyze the different characteristic features of periodic, and aperiodic signals using CRO.
- Use CTs and PTs for measurement of very large currents and high voltages

TEXT BOOKS:


REFERENCE BOOKS:


UNIT 1

INTRODUCTION TO ELECTRICAL MEASUREMENTS

DEFLECTING, CONTROLLING AND DAMPING TORQUES

Deflecting Torque:
The deflecting torque is produced by making use of one of the magnetic, heating, chemical, electrostatic and electromagnetic induction effects of current or voltage and causes the moving system of the instrument to move from its zero position when the instrument is connected in an electrical circuit to measure the electrical quantity. The method of producing this torque depends upon the type of the instrument.

Controlling Torque:
Under the action of deflecting torque, the pointer will continue to move indefinitely and shall be independent of the value of electrical quantity to be measured. This controlling torque should oppose the deflecting torque and should increase with the deflection of the moving system so that the pointer is brought to rest at a position when the two opposing torques are equal. The controlling torque performs the following functions:

- It opposes the deflecting torque and increases with the deflection of the moving system. It thus limits the movement of the pointer so that the magnitude of deflection is always the same for the given value of electrical quantity to be measured.
- It brings the pointer to the zero position when the deflecting torque is removed. If it were not provided, the pointer once deflected would not return to zero position on removing the deflecting torque.

The controlling torque is provided either by spring control or gravity control. A hair spring usually of phosphor bronze is attached to the moving system of the instrument. With the movement of the pointer, the spring is twisted in opposite direction. This twist in spring produces restoring torque. Gravity control is obtained by attaching a small adjustableweight to the moving system. Such that the two exert torques in opposite directions.

Spring Control:
Two hair springs are attached to the moving system which exerts controlling torque. To employ spring control to an instrument, following requirements are essential.

- The spring should be non-mechanical stress.
- The spring should be free from mechanical stress.
- They should have small resistance and sufficient cross sectional area.
- They should have low resistance temperature coefficient.
Springs are made up of non magnetic materials like silicon bronze, hard rolled silver or copper, platinum silver and german silver. For most of the instruments phosphor bronze spiral springs are provided. Flat spiral springs are used in almost all indicating instruments.

The inner end of the spring is attached to the spindle while the outer end is attached to a lever or arm which is actuated by a set of screw mounted at the front of the instrument. So zero sitting can be easily done. The controlling torque provided by the instrument is directly proportional to the angular deflection of the pointer.

**Advantages of Spring Control:**
1. Scale is uniform.
2. The readings are taken very accurately.
3. The system need not be in vertical position. It can be used in any position.
4. This control is mostly used when compared to gravity control.

**Disadvantages of Spring Control:**
1. The controlling torque is fixed.
2. The performance is temperature dependent.
3. It is simple, rigid but costlier.

**Gravity Control**
This type of control consists of a small weight attached to the moving system whose position is adjustable. This weight produces a controlling torque due to gravity. This weight is called control weight. The control system is as shown in the figure. At the zero position of the pointer, the controlling torque is zero. The control weights acts at a distance 'l' from the center.
Gravity Control

Advantages of Gravity Control:
1. The performance is not time dependent.
2. It is simple and cheap.
3. Controlling torque can be varied by adjusting the position of the control weight.
4. The performance is not temperature dependent.

Disadvantages of Gravity Control:
1. Scale is non-uniform. So readings are not accurate.
2. The system is used in vertical position only and must be properly levelled. Otherwise, it may cause serious errors in the measurements.
3. As delicate and proper levelling required, in general it is not used for indicating instruments and portable instruments.

DAMPING TORQUE:

If the moving system is acted upon by deflecting and controlling torques alone, then the pointer due to its inertia will oscillate about final position before coming to rest. These oscillations are undesirable and must be prevented. In order to avoid these oscillations of the pointer and to bring it quickly to its deflected position, damping torque is provided which opposes the movement of the pointer and operates only when the system is moving.

The damping torque should have a magnitude that the pointer quickly comes to its final steady position. If the system is *underdamped*, the moving system will oscillate about the final steady position with a decreasing amplitude. When the moving system moves rapidly but smoothly to
its final steady position, the system is said to be critically damped. If the damping torque is more than what is required for critical damping is called over-damped. The figure below shows the way an underdamped, an overdamped and critically damped system moves to its final steady position.

The methods for producing damping torque are:

1. Air friction damping
2. Fluid friction damping
3. Eddy current damping

**1. Air Friction damping:**
The air friction damping device is as shown in the figure below. The arrangement consists of a light aluminium piston which is attached to the moving system. This piston moves in a fixed air chamber which is closed at one end. The clearance between piston and the chamber walls is uniform throughout and is very small. When there are oscillations the piston moves into and out of air chamber. When the piston moves inside the chamber, the air inside is compressed and pressure of air, thus builds up, opposes the motion of piston and hence the whole moving system. When the piston moves out of the air chamber, pressure in the closed space falls, and the pressure on the open side of piston is greater than on the other side. Thus there is again an opposition to motion.
2. Fluid Friction damping:
This form of damping is similar to air friction damping. Oil is used in place of air and as the viscosity of oil is greater, the damping force is correspondingly greater.

A disc is attached to the moving system as shown in the figure, this disc dips into an oil pot and is completely submerged in oil. When the moving system moves, the disc moves in oil and a frictional drag is produced. This frictional drag always opposes the motion.

Advantages of fluid friction damping:
- Due to more viscosity of fluid, more damping is provided.
- The oil can also be used for insulation purposes.
- Due to upthrust of oil, the load on the bearings are reduced, thus reducing the frictional errors.

Disadvantages of fluid friction damping:
- These are used only for instruments in vertical position.
- Due to oil leakage, the instruments cannot be kept clean.
3. **Eddy Current damping:**

This is the most efficient way of providing damping. It is based on Faraday's law and Lenz's law. When a conductor moves in a magnetic field cutting the flux, the e.m.f gets induced in it. And direction of this e.m.f is so as to oppose the cause producing it.

![Eddy current damping](image)

In this method aluminum disc is connected to the spindle. The arrangement of disc is such that when it rotates, it cuts the magnetic flux lines of a permanent magnet. The arrangement is as shown in the figure.

When the pointer oscillates, aluminum disc rotates under the influence of magnetic field of damping magnet. So disc cuts the flux which causes an induced e.m.f in the disc. The disc is a closed path hence induced e.m.f circulate current through the disc called eddy current. The direction of such eddy current is so as to oppose the cause producing it. The cause is relative motion between disc and field. Thus it produces an opposing torque so as to reduce the oscillations of pointer. This brings pointer to rest quickly. This is most effective and efficient method of damping.
**PERMANENT MAGNET MOVING COIL INSTRUMENTS:**

These instruments are employed either as ammeters or voltmeters and can be used for d.c work only.

**Principle of Permanent magnet moving coil instruments:**
This type of instrument is based on the principle that when a current carrying conductor is placed in a magnetic field, mechanical force acts on the conductor. The coil placed in magnetic field and carrying operating current is attached to the moving system. With the movement of the coil the pointer moves over the scale.

**Construction of Permanent magnet moving coil instruments:**
It consists of a powerful permanent magnet with soft iron pieces and light rectangular coil of many turns of fine wire wound on aluminium former inside which is an iron core as shown in fig. The purpose of coil is to make the field uniform. The coil is mounted on the spindle and acts as the moving element. The current is led into and out of the coil by means of the to control hair springs, one above and the other below the coil. The springs also provides the controlling torque. Eddy current damping is provided by aluminium former.

![Permanent magnet moving coil instrument](image)

**Working of Permanent magnet moving coil instruments:**
When the instrument is connected in the circuit, operating current flows through the coil. This current coil is placed in the magnetic field produced by the permanent magnet and therefore, mechanical force acts on the coil. As the coil attached to the moving system, the pointer moves over the scale.

It maybe noted that if current direction is reversed, the torque will also be reversed since the direction of the field of permanent magnet is same. Hence, the pointer will move in the opposite direction, i.e it will go on the wrong side of zero. In other words, these instruments work only when current in the circuit is passed in a definite direction i.e. for d.c circuit only.
It is worthwhile to mention here that such instruments are called permanent magnet moving coil instruments because a coil moves in the field of a permanent magnet.

**Deflecting torque of Permanent magnet moving coil instruments:**
When current is passed through the coil, a deflection torque is produced due to the reaction between permanent magnetic field and the magnetic field of the coil as shown in the fig.

Let $B =$ flux density in the air gap between the magnetic poles and iron core.

$l =$ active length of the coil side in meters.

$N =$ number of turns of coil.

If a current of $i$ amperes flows in the coil in the direction shown, then force $F$ acting on each coil side is given by

$$F = \frac{BI}{N} \text{ newtons}$$

$$T_d = \text{Force} \times \text{perpendicular distance}$$

$$= F \times 2r \text{ newton - meters}$$

where $r$ is the distance of coil side from the axis of rotation in meters

$$T_d = NBII \times 2r \text{ newton - meters}$$

If $A$ ($= l \times 2r$) is the surface area of the coil, then

$$T_d = NBIA \text{ newton - meters}$$

Deflection torque = Ampere turns on coil $(NI)$ * Area of coil $(A)$ * flux density $(B)$

$$T_d \text{ proportional } I$$

Since the control by springs, therefore controlling torque is proportional to the angle of deflection i.e

$$T_c \text{ proportional } I$$

The pointer will come to rest at a position when,

$$T_d = T_c$$

Deflection proportional $I$
Thus, the deflection is directly proportional to the operating current. Therefore, such instruments have uniform scale.

**Advantages of Permanent magnet moving coil instruments:**
1. uniform scale.
2. Very effective eddy current damping
3. Power consumption is low.
4. No hysteresis loss
5. As working field is very strong, therefore, such instruments are not affected by stray fields.
6. Such instruments require small operating current.
7. Very accurate and reliable.

**Disadvantages of Permanent magnet moving coil instruments:**
1. Such instruments cannot be used for a.c measurements.
2. Costlier as compared to moving iron instruments.
3. Some errors are caused due to the ageing of control springs and the permanent magnet.

**MOVING IRON INSTRUMENT:**

Moving iron instruments are of two types:
1. Attraction type Moving iron instrument
2. Repulsion type Moving iron instrument

**ATTRACTION TYPE MOVING IRON INSTRUMENT:**

**Principle of Attraction type Moving iron instrument:**
These instruments are based on the principle that when an unmagnetised soft iron piece is placed in the magnetic field of a coil, then the piece is attracted towards the coil. The moving system of the instrument is attached to the soft iron piece and the operating current is passed through a coil placed near it. The operating current sets up a magnetic field which attracts the iron piece and thus moves the pointer over the scale.
**Construction of Attraction type Moving iron instrument:**

It consists of a hollow cylindrical coil or solenoid which is kept fixed as shown in the fig. An oval shaped soft iron pieces is attached to the spindle in such a way that it can move in or out of the coil. The pointer is attached to the spindle so that it is deflected with the motion of the soft iron piece. The controlling torque on the moving system is provided by spring control method while damping is provided by air friction.

![Attraction type Moving iron instrument](image)

**Attraction type Moving iron instrument**

**Working of Attraction type Moving iron instrument:**

When the instrument is connected in the circuit, the operating current flows through the coil. The current sets up magnetic field in the coil. In other words, the coil behaves like a magnet and, therefore, it attracts the soft iron piece towards it. The result is that the pointer attached to the moving system moves from zero position.

If current in the coil is reversed, the direction of magnetic field also reverses and so does the magnetism produced in soft iron piece. Therefore, the direction of deflecting torque remains unchanged. It follows, therefore, that such instruments can be used for both D.C as well as A.C work.

**Deflecting torque of Attraction type Moving iron instrument:**

Field strength $h$ produced by the coil. $m1$

Pole strength $m$ developed by the piece $m2$

\[
F \propto m1 \cdot m2 \cdot H \\
F \propto (\text{field strength})^2 \\
Td \propto F \propto H^2 \\
H \propto I \\
Td \propto I^2
\]

If the controlling torque is provided by the springs,

\[
Tc \propto \text{deflection} \Phi
\]
In the steady state position of deflection,

\[ T_d = T_c \]

Deflection proportional \( I^2 \)
Deflection proportional \( I_{rms} \)

Scale of such instruments is non-uniform, being crowded in the beginning. In order to make the scale of such instruments uniform, suitably shaped iron piece is used.

**REPULSION TYPE MOVING IRON INSTRUMENTS:**

**Principle of Repulsion type moving iron instruments:**

These instruments are based on the principle of repulsion between the two iron pieces similarly magnetized.

**Construction of Repulsion type moving iron instruments:**

It consists of a fixed cylindrical hollow coil which carries operating current. Inside the coil, there are two soft iron pieces or vanes, one of which is fixed and other is movable. The fixed iron piece is attached to the coil whereas the movable iron piece is attached to the pointer shaft. Under the action of deflecting torque, the pointer attached to the moving system moves over the scale. The controlling torque is produced by spring control method and damping torque by air friction damping.

![Image of Repulsion Type Moving Iron Instrument](image)

**Working of Repulsion type moving iron instruments:**

When the instrument is connected in the circuit, current flows through the coil. This current sets up magnetic field in the coil. The magnetic field magnetizes both iron pieces in the same direction i.e. both pieces become similar magnets and hence they repel each other. Due to this force of repulsion only movable iron piece moves as the other piece is fixed and cannot move. The result is that the pointer attached to the moving system moves from zero position.
Repulsion type moving iron instruments:

**Deflecting torque of Repulsion type moving iron instruments:**
The deflecting torque results due to the repulsion between the similarly magnetized iron pieces. If two pieces develop pole strengths $m_1$ and $m_2$ respectively, then,

Instantaneous deflecting torque proportional to repulsive force

$$\text{proportional } m_1.m_2$$

Since pole strengths developed are proportional to $H$, therefore

Instantaneous deflecting torque, $T_d$ proportional $H^2$

Assuming constant permeability, $h$ proportional current $i$ through the coil

$$\text{proportional } i^2$$

Controlling torque provided by springs, $T_c$ proportional deflection

In the steady position of deflection,

$$T_d = T_c$$

Deflection proportional to $i^2$

$$\text{proportional } I^2$$

Proportional $\text{Irms}^2$

Since deflection is proportional to square of current through the coil, therefore, scale of such instruments in non-uniform being crowded in the beginning. However, scale of such instruments can be made uniform by using tongue shaped iron pieces.

**Advantages of moving iron instruments:**

1. These are cheap, robust and simple in construction.
2. The instruments can be used for both A.C as well as D.C circuits.

3. These instruments have high operating torque.

4. These instruments are reasonable accurate.

**Disadvantages of moving iron instruments:**
1. Such instruments have non-uniform scale.

2. These instruments are not very sensitive.

3. Errors are introduced due to changes in frequency in case of a.c measurements.

4. Higher power consumption.

**Dynamometer type instruments:**
These instruments are the modified form of permanent magnet moving coils type. Here operating field is produced by a permanent but by another fixed coil. The moving system and the control system are similar to those of permanent magnet type. Such instruments can be used for both a.c and d.c circuits. They can be used as ammeters and voltmeters but are generally used as wattmeters.

**Principle of Dynamometer type instruments:**
These instruments are based on that principle the mechanical force exists between the current carrying conductors.

**Construction of Dynamometer type instruments:**
A dynamometer type instrument as shown in fig essentially consists of a fixed coil and a moving coil. The fixed coil is split into two equal parts which are placed close together and parallel to each other. The moving coil is pivoted in between the two fixed coils. The fixed and moving coils may be excited separately or they may be connected in series depending upon the use to which the measurement is put. The moving coil is attached to the moving system so that under the action of deflecting torque, the pointer moves over the scale.
The controlling torque is provided by two springs which also serve the additional purpose of leading the current into and out of the moving coil. Air friction damping is provided in such instruments.

**Working of Dynamometer type instruments:**
When instrument is connected in the circuit, operating currents flow through the coils. Due to this, mechanical force exists between the coils. The result is that the moving coil moves the pointer over the scale. The pointer comes to rest at a position where deflecting torque is equal to the controlling torque.

by reversing the current, the field due to fixed coils is reversed as well as the current in the moving coil, so that the direction of deflecting torque remains unchanged. Therefore, such instruments can be used for both d.c and a.c measurements.

**Deflecting torque of Dynamometer type instruments:**
Let
If = current through fixed coil
Im = current through moving coil
Since If = Im because the fixed and coils are in series,

\[ T_d = I^2 \]

Since the control is by springs, therefore,

controlling torque is proportional to the angle of deflection

\[ T_c \propto \text{deflection} \]

The pointer will come to rest at a position when \( T_d = T_c \)
we get deflection proportional \( I^2 \)

It is clear that deflection of the pointer is directly proportional to the square of the operating current. Hence, the scales of these instruments is non-uniform being crowded in their lower parts and spread out at the top.

**Advantages of Dynamometer type instruments:**
1. These instruments can be used for both a.c and d.c measurements.
2. Such instruments are free from hysteresis and eddy current errors.

**Disadvantages of Dynamometer type instruments:**
1. Since torque / weight ratio is small, therefore, such instruments have frictional errors which reduce sensitivity.
2. Scale is not uniform.
3. A good amount of screening of the instruments are required to avoid the effect of stray fields.
4. These instruments are costlier than types and, therefore, they are rarely used as ammeters and voltmeters.
**CATHODE RAY OSCILLOSCOPE:**

- The cathode ray oscilloscope is an electronic instrument that presents a high fidelity graphical display of the rapidly changing voltage at its input terminals. The cathode ray oscilloscope is probably the most versatile and useful instrument available for signal measurement.

- Unlike meters, which only allow the user to measure amplitude information, the oscilloscope allows the user to view the instantaneous voltage versus time such displayed plot of the signal can be used for various measurements like peak voltage, frequency, phase, time period, rise time etc. It can also indicate the nature and magnitude of noise that may be corrupting the measurement signal. The more expensive models can measure signals at frequencies up to 500 MHz and even the cheapest models can measure signals at frequencies up to 20 MHz. One particularly strong merit of the oscilloscopes is its high input impedance, typically 1M, which means that the instrument has a negligible loading effect in most measurement situations. As a test instrument, it is often required to measure voltages whose frequency and magnitude are totally unknown.

- However, it is not a particularly accurate instrument and is best used where only an approximate measurement is required. Further disadvantages of oscilloscopes include their fragility and their moderately high cost.

- There are two main classes of oscilloscopes: analog oscilloscopes and digital oscilloscopes.

- A simplified block diagram of an analog oscilloscope is shown in below figure.

![Analog oscilloscope block diagram](image)

- The display section of the cathode ray oscilloscope has two inputs to it namely vertical input and horizontal input. The signals applied to these inputs are driven to corresponding deflection plates and control the position of the electron beam that plots the waveform on the screen. There are two types of plots that can be displayed based on mode of operation of oscilloscope.
Important measurements that can be made by cathode ray oscilloscope:
2. Measurement of power and power factor.
5. Tracing of hysterisis loop for magnetic material.
7. Fault testing of windings of electrical machines.
8. Determination of characteristics of thermionic values.

Construction of Cathode Ray Oscilloscope

The main part of cathode ray oscilloscope is cathode ray tube which is also known as the heart of cathode ray oscilloscope.

Let us discuss the construction of cathode ray tube in order to understand the construction of cathode ray oscilloscope. Basically the cathode ray tube consists of five main parts and these main parts are written below:
1. Electron gun.
2. Deflection plate system.
3. Fluorescent screen.
5. Base.

Now we discuss each of the above part in detail:

Electron Gun: It is the source of accelerated, energized and focused beam of electrons. It consists of six parts namely heater, a cathode, a grid, a pre-accelerating anode, a focusing anode and an accelerating anode. In order to obtain the high emission of electrons the layer of barium oxide (which is deposited on the end of
cathode) is indirectly heated at moderate temperature. The electrons after this passes through a small hole called control grid which is made up of nickel. As the name suggests the control grid with its negative bias, controls the number of electrons or indirectly we can say the intensity of emitted electrons from cathode. After passing through the control grid these electrons are accelerated with the help of pre-accelerating and accelerating anodes. The pre-accelerating and accelerating anodes are connected to a common positive potential of 1500 volts.

Now after this the function of the focusing anode is to focus the beam of the electrons so produced. The focusing anode is connected to adjustable voltage 500 volts. Now there are two methods of focusing the electron beam and are written below:

1. Electrostatic focusing.
2. Electromagnetic focusing.

Here we will discuss electrostatic focusing method in detail.

**Electrostatic Focusing** We know that the force on an electron is given by \(-qE\), where \(q\) is the charge on electron (\(q = 1.6 \times 10^{-19}\) C), \(E\) is the electric field intensity and negative sign shows that the direction of force is in opposite direction to that of electric field. Now we will this force to defect the beam of electrons coming out of electron gun. Let us consider two cases:

**Case One** In this case we are having two plates A and B as shown in the figure.

The plate A is at potential +E while the plate B is at potential −E. The direction of electric field is from A plate to plate B at right angle to the surfaces of the plate. The equipotential surfaces are also shown in the diagram which is perpendicular to the direction of electric field. As the beam of electron passes through this plate system, it deflects in the opposite direction of electric field. The deflection angle can be easily varied by changing the potential of the plates.

**Case Second** Here we have two concentric cylinders with a potential difference applied between them as shown in the figure.
The resultant direction of electric field and the equipotential surfaces are also shown in the figure. The equipotential surfaces are marked by the dotted lines which are curved in shape. Now here we are interested in calculating the deflection angle of electron beam when it passes through this curved equipotential surface. Let us consider the curved equipotential surface S as shown below. The potential on the right side of the surface is +E while the potential on the left side of the surface –E. When a beam of electron is incident at angle A to the normal then it deflects by angle B after passing through the surface S as shown in the figure given below. The normal component of velocity of the beam will increase as force is acting in s direction normal to the surface. It means that the tangential velocities will remain same, so by equating the tangential components we have \( V_1 \sin(A) = V_2 \sin(B) \), where \( V_1 \) is the initial velocity of the electrons, \( V_2 \) is the velocity after passing through the surface. Now we have relation as \( \sin(A)/\sin(B) = V_2 / V_1 \). We can from the above equation see that there is bending of the electron beam after passing through the equipotential surface. Therefore this system is also called focusing system. **Electrostatic Deflection** In order to find out the expression for the deflection, let us consider a system as shown below:
In the above system we have two plates A and B which are at potential +E and 0 respectively. These plates are also called deflection plates. The field produced by these plates is in the direction of positive y axis and there is no force along the x-axis. After deflection plates we have screen through which we can measure net deflection of the electron beam. Now let us consider a beam of electron coming along the x-axis as shown in the figure. The beam deflects by angle A, due presence of electric field and deflection is in the positive direction of y axis as shown in the figure. Now let us derive an expression for deflection of this beam. By the conservation of energy, we have loss in potential energy when the electron moves from cathode to accelerating anode should be equal to gain in kinetic energy of electron. Mathematically we can write,

\[ eE = \frac{1}{2}mv^2 \]  \hspace{1cm} (1)

Where, e is the charge on electron, E is the potential difference between the two plates, m is the mass of electron, and v is the velocity of the electron. Thus, eE is loss in potential energy and 1/2mv^2 is the gain in kinetic energy. From equation (1) we have velocity \( v = (2eE/m)^{1/2} \). Now we have electric field intensity along the y axis is E/d, therefore force acting along the y axis is given by \( F = eE/d \) where d is the separation.
between the two deflection plates. Due to this force the electron will deflect along the y axis and let the deflection along y axis be equal to D which is marked on the screen as shown in the figure. Due to the force F there is net upward acceleration of the electron along positive y axis and this acceleration is given by \( \frac{Ee}{(d \times m)} \). Since the initial velocity along positive y direction is zero therefore by equation of motion we can write the expression of displacement along y axis as,

\[
y = \frac{1}{2} \left( \frac{Ee}{m \times d} \right) \times t^2 \quad \text{..................( 2 )}
\]

As the velocity along the x direction is constant therefore we can write displacement as,

\[
x = u \times t \quad \text{..................( 3 )}
\]

Where, \( u \) is velocity of electron along x axis. From equations 2 and 3 we have

\[
y = \frac{1}{2} \left( \frac{eE}{mu^2} \right) \times x^2 \quad \text{..........................( 4 )}
\]

Which is the equation of trajectory of the electron. Now on differentiating the equation 4 we have slope i.e.

\[
\frac{dy}{dx} = \frac{eEl}{mu^2}
\]

Where, \( l \) is the length of the plate. Deflection on the screen can be calculated as,

\[
D = L \times \frac{dy}{dx}
\]

Distance \( L \) is shown in the above figure. Final expression of D can be written as,

\[
D = \frac{LlE}{2dE}
\]

From the expression of deflection, we calculate deflection sensitivity as,

\[
\frac{D}{E} = \frac{Ll}{2dE}
\]

**Graticule:** These are the grid of lines whose function is to serve as a scale when the **cathode ray oscilloscope** is used for the amplitude measurements. There are three types of graticules and they are written below:

1. **Internal Graticule:** Internal graticule as name suggests deposited on the internal surface of the cathode ray tube face plate. There is no problem of parallax errors but we cannot change internal graticules as they are fixed.
2. **External graticule:**
Given below is the circuit diagram of cathode ray oscilloscope:

Basic Circuit Diagram of Cathode Ray Oscilloscope

Now we will study the basic circuit diagram of cathode ray oscilloscope under the following main parts.

1. **Vertical Deflection System**: The input signal for examining are fed to the vertical deflection system plates with the help of input attenuator and a number of amplifier stages. The main function of these amplifiers is to amplify the weak signals so that the amplified signal can produce the desirable signals.

2. **Horizontal Deflection System**: Like the vertical system horizontal system also consists of horizontal amplifiers to amplify the weak input voltage signals but in contrast to vertical deflection system, horizontal deflection plates are fed by a sweep voltage that provides a time base as shown above. As shown in the circuit diagram, the saw tooth sweep generator is triggered by the synchronizing amplifier when the sweep selector switch is in the internal position and thus the triggered saw tooth generator gives input to the horizontal amplifier by following this mechanism. Now there are four types of sweeps:
MEASUREMENT OF RESISTANCE

Resistance is one of the most basic elements encountered in electrical and electronics engineering. The value of resistance in engineering varies from very small value like, resistance of a transformer winding, to very high values like, insulation resistance of that same transformer winding. Although a multimeter works quite well if we need a rough value of resistance, but for accurate values and that too at very low and very high values we need specific methods. In this article we will discuss various methods of resistance measurement. For this purpose we categories the resistance into three classes-

- MEASUREMENT OF LOW RESISTANCE (<1Ω)
  
  The major problem in measurement of low resistance values is the contact resistance or lead resistance of the measuring instruments, though being small in value is comparable to the resistance being measured and hence causes serious error.
  
  The methods employed for measurement of low resistances are:
  
  - Kelvin’s Double Bridge Method
  - Potentiometer Method
  - Ducter Ohmmeter.

  **KELVIN’S DOUBLE BRIDGE**

  Kelvin’s double bridge is a modification of simple Wheatstone bridge. Figure below shows the circuit diagram of Kelvin’s double bridge.
As we can see in the above figure there are two sets of arms, one with resistances P and Q and other with resistances p and q. R is the unknown low resistance and S is a standard resistance. Here r represents the contact resistance between the unknown resistance and the standard resistance, whose effect we need to eliminate. For measurement we make the ratio P/Q equal to p/q and hence a balanced Wheatstone bridge is formed leading to null deflection in the galvanometer. Hence for a balanced bridge we can write

\[ E_{od} = E_{nc} \]

Or,
\[ \left\{ \frac{P}{P + Q} \right\} E_{ab} = I \left[ R + \frac{p}{p + q} \left\{ \frac{r(p + q)}{p + q + r} \right\} \right] \quad \ldots \ldots (1) \]

Where, \[ E_{ob} = I \left[ R + S + \frac{p}{p + q} \left\{ \frac{r(p + q)}{p + q + r} \right\} \right] \quad \ldots \ldots (2) \]

Putting eqn 2 in 1 and solving and using \( P/Q = p/q \), we get-

\[ R = \frac{PS}{Q} \]

Hence we see that by using balanced double arms we can eliminate the contact resistance completely and hence error due to it. To eliminate another error caused due to thermo-electric emf, we take another reading with battery connection reversed and finally take average of the two readings. This bridge is useful for resistances in range of 0.1µΩ to 1.0 Ω.

**MEASUREMENT OF MEDIUM RESISTANCE (1Ω - 100KΩ)**

Following are the methods employed for measuring a resistance whose value is in the range 1Ω - 100kΩ -

- Ammeter-Voltmeter Method
- Wheatstone Bridge Method
- Substitution Method
- Carey- Foster Bridge Method
- Ohmmeter Method

**WHEATSTONE BRIDGE METHOD**

This is the simplest and the most basic bridge circuit used in measurement studies. It mainly consists of four arms of resistance P, Q; R and S. R is the unknown resistance under experiment, while S is a standard resistance. P and Q are known as the ratio arms. An EMF source is connected between points a and b while a galvanometer is connected between points c and d.
A bridge circuit always works on the principle of null detection, i.e. we vary a parameter until the detector shows zero and then use a mathematical relation to determine the unknown in terms of varying parameter and other constants. Here also the standard resistance, S is varied in order to obtain null deflection in the galvanometer. This null deflection implies no current from point c to d, which implies that potential of point c and d is same. Hence

\[ I_1 P = I_2 R \quad \ldots \quad (4) \]

Also, \( I_1 = I_3 = \frac{E}{(P + Q)} \) and \( I_2 = I_4 = \frac{E}{(R + S)} \quad \ldots \quad (5) \)

Combining the above two equations we get the famous equation –

\[ R = \frac{P}{Q} S \]

**MEASUREMENT OF HIGH RESISTANCE (>100KΩ)**

Following are few methods used for measurement of high resistance values –

- Loss of Charge Method
- Megger
- Megohm bridge Method
- Direct Deflection Method
LOSS OF CHARGE METHOD
In this method we utilize the equation of voltage across a discharging capacitor to find the value of unknown resistance R. Figure below shows the circuit diagram and the equations involved are-

\[ v = V e^{-\frac{t}{CR}} \]
\[ R = \frac{0.4343t}{C \log_{10}V/v} \]

However the above case assumes no leakage resistance of the capacitor. Hence to account for it we use the circuit shown in the figure below. \( R_1 \) is the leakage resistance of C and R is the unknown resistance. We follow the same procedure but first with switch \( S_1 \) closed and next with switch \( S_1 \) open. For the first case we get

\[ R' = \frac{0.4343t}{C \log_{10}V/v} \]
\[ \text{Where, } R' = \frac{RR_1}{R + R_1} \]

For second case with switch open we get

\[ R_1 = \frac{0.4343t}{C \log_{10}V/v} \]

Using \( R_1 \) from above equation in equation for \( R' \) we can find R.

MAXWELL BRIDGE
A Maxwell bridge is a modification to a Wheatstone bridge used to measure an unknown inductance (usually of low Q value) in terms of calibrated resistance and inductance or resistance and capacitance. When the calibrated components are a parallel resistor and capacitor, the bridge is known as a Maxwell-Wien bridge. It is named for James C. Maxwell, who first described it in 1873.
It uses the principle that the positive phase angle of an inductive impedance can be compensated by the negative phase angle of a capacitive impedance when put in the opposite arm and the circuit is at resonance; i.e., no potential difference across the detector (an AC voltmeter or ammeter)) and hence no current flowing through it. The unknown inductance then becomes known in terms of this capacitance.

With reference to the picture,

\[ R_1 \text{ and } R_4 \text{ are known fixed resistances} \]
\[ R_2, C_2 \text{ are known variable resistance, capacitances} \]
\[ \text{variable entities. } R_2 \text{ and } C_2 \text{ are adjusted until the bridge is balanced.} \]
\[ R_3 \text{ and } L_3 \text{ can then be calculated based on the values of the other components} \]
\[ R_3 = \frac{R_1 \cdot R_4}{R_2} \]
\[ L_3 = R_1 \cdot R_4 \cdot C_2 \]

Anderson's Bridge

The main disadvantage of using Hay's bridge and Maxwell bridge is that, they are unsuitable of measuring the low quality factor. However Hay's bridge and Maxwell bridge are suitable for measuring accurately high and medium quality factor respectively. So, there is need of bridge which can measure low quality factor and this bridge is modified Maxwell's bridge and known as Anderson's bridge. Actually this bridge is the modified Maxwell inductor capacitance bridge. In this bridge double balance can obtained by fixing the value of capacitance and changing the value of electrical resistance only.
In this circuit the unknown inductor is connected between the point a and b with electrical resistance $r_1$ (which is pure resistive). The arms bc, cd and da consist of resistances $r_3$, $r_4$ and $r_2$ respectively which are purely resistive. A standard capacitor is connected in series with variable electrical resistance $r$ and this combination is connected in parallel with cd. A supply is connected between b and e.

Now let us derive the expression for $l_1$ and $r_1$:

At balance point, we have the following relations that holds good and they are:

$$i_1 = i_3 \text{ and } i_2 = i_c + i_4$$

Now equating voltages drops we get,

$$i_1 . r_3 = \frac{i_c}{j\omega C} \quad \text{......(1)}$$

$$i_1 . (R_1 + r_1 + j\omega l_1) = i_2 . r_2 + i_c . r \quad \text{......(2)}$$

$$i_c \left( r + \frac{1}{j\omega C} \right) = (i_2 - i_c) r_4 \quad \text{......(3)}$$

Putting the value of $i_c$ in above equations, we get

$$i_1 (r_1 + R_1 + j\omega l_1) = i_2 r_2 + ji_1 \omega C r_3 r$$

$$i_1 (r + r_1 + j\omega l_1 - j\omega C r_3 r) = i_2 r_2 \quad \text{......(4)}$$

or we have

$$i_1 (j\omega C r_3 r + j\omega C r_3 r_4 + r_3) = i_2 r_4 r_2 \quad \text{......(5)}$$

On equating (4) and (5) and separating the real and imaginary parts are have,

$$r_1 = \frac{r_2 r_3}{r_4} - R_1 r_2 \quad \text{......(6)}$$

$$\text{and } l_1 = \frac{C . r_3}{r_4} [r(r_4 + r_2) + r_2 r_4] r_2 \quad \text{......(7)}$$

WORKING
At first set the signal generator frequency at audible range. Now adjust \( r_1 \) and \( r \) such that phones gives a minimum sound. Measure the values of \( r_1 \) and \( r \) (obtained after these adjustments) with the help of multimeter. Use the formula that we have derived above in order to find out the value of unknown inductance. The experiment can be repeated with the different value of standard capacitor.

Phasor Diagram of Anderson's Bridge

Let us mark the voltage drops across ab, bc, cd and ad as \( e_1, e_2, e_3 \) and \( e_4 \) as shown in figure above.

Phasor Diagram for Andersons

Here in the phasor diagram of Anderson's bridge, we have taken \( i_1 \) as reference axis. Now \( i_c \) is perpendicular to \( i_1 \) as capacitive load is connected at ec, \( i_4 \) and \( i_2 \) are lead by some angle as shown in figure. Now the sum of all the resultant voltage drops i.e. \( e_1, e_2, e_3 \) and \( e_4 \) is equal to \( e \), which is shown in phasor diagram. As shown in the phasor diagram of Anderson's bridge the resultant of voltages drop \( i_1 (R_1 + r_1) \) and \( i_1, \omega l_1 \) (which is shown perpendicular to \( i_1 \)) is \( e_1 \). \( e_2 \) is given by \( i_2, r_2 \) which makes angle 'A' with the reference axis. Similarly, \( e_4 \) can be obtained by voltage drop \( i_4, r_4 \) which is making angle 'B' with reference axis.

Advantages of Anderson's Bridge

1. It is very easy to obtain the balance point in Anderson's bridge as compared to Maxwell bridge in case of low quality factor coils.
2. There is no need of variable standard capacitor is required instead of thin a fixed value capacitor is used.
3. This bridge also gives accurate result for determination of capacitance in terms of inductance.
Disadvantages of Anderson's Bridge
1. The equations obtained for inductor in this bridge is more complex as compared to Maxwell's bridge.
2. The addition of capacitor junction increases complexity as well as difficulty of shielding the bridge.

De Sauty Bridge
This bridge provide us the most suitable method for comparing the two values of capacitor if we neglect dielectric losses in the bridge circuit. The circuit of De Sauty's bridge is shown below

Battery is applied between terminals marked as 1 and 4. The arm 1-2 consists of capacitor \( c_1 \) (whose value is unknown) which carries current \( i_1 \) as shown, arm 2-4 consists of pure resistor (here pure resistor means we assuming it non inductive in nature), arm 3-4 also consists of pure resistor and arm 4-1 consists of standard capacitor whose value is already known to us. Let us derive the expression for capacitor \( c_1 \) in terms of standard capacitor and resistor. At balance condition we have,

\[
\frac{1}{j\omega c_1} \times r_4 = \frac{1}{j\omega c_2} \times r_3
\]

It implies that the value of capacitor is given by the expression

\[
c_1 = c_2 \times \frac{r_4}{r_3}
\]

In order to obtain the balance point we must adjust the values of either \( r_3 \) or \( r_4 \) without disturbing any other element of the bridge. This is the most efficient method of comparing the two values of capacitor if all the dielectric losses are neglected from the circuit.
Now let us draw and study the phasor diagram of this bridge. Phasor diagram of **De Sauty bridge** is shown below:

Let us mark the current drop across unknown capacitor as $e_1$, voltage drop across the resistor $r_3$ be $e_3$, voltage drop across arm 3-4 be $e_4$ and voltage drop across arm 4-1 be $e_2$. At balance condition the current flows through 2-4 path will be zero and also voltage drops $e_1$ and $e_3$ be equal to voltage drops $e_2$ and $e_4$ respectively.

In order to draw the phasor diagram we have taken $e_3$ (or $e_4$) reference axis, $e_1$ and $e_2$ are shown at right angle to $e_1$ (or $e_2$). Why they are at right angle to each other? Answer to this question is very simple as capacitor is connected there, therefore phase difference angle obtained is $90^\circ$.

Now instead of some advantages like bridge is quite simple and provides easy calculations, there are some disadvantages of this bridge because this bridge give inaccurate results for imperfect capacitor (here imperfect means capacitors which not free from dielectric losses). Hence we can use this bridge only for comparing perfect capacitors.

Here we interested in modify the **De Sauty's bridge**, we want to have such a kind of bridge that will gives us accurate results for imperfect capacitors also. This modification is done by Grover. The modified circuit diagram is shown below:

Here Grover has introduced electrical resistances $r_1$ and $r_2$ as shown in above on arms 1-2 and 4-1 respectively, in order to include the dielectric losses. Also he has connected resistances $R_1$ and $R_2$ respectively in the arms 1-2 and 4-1. Let us derive the expression capacitor $c_1$ whose value is unknown to us. Again we connected standard capacitor on the same arm 1-4 as we have done in **De Sauty's bridge**. At balance point on equating the voltage drops we have:

$$\left( R_1 + r_1 + \frac{1}{j\omega c_1} \right) r_4 = \left( R_2 + r_2 + \frac{1}{j\omega c_2} \right) r_3 \cdots \cdots \cdots \cdots \cdots (1)$$
On solving above equation we get:
\[
\frac{c_1}{c_2} = \frac{R_2 + r_2}{R_1 + r_1} = r_4r_3
\]
This the required equation. By making the phasor diagram we can calculate dissipation factor. Phasor diagram for the above circuit is shown below.

Let us mark $\delta_1$ and $\delta_2$ be phase angles of the capacitors $c_1$ and $c_2$ capacitors respectively. From the phasor diagram we have $\tan(\delta_1) = \text{dissipation factor} = \omega c_1 r_1$ and similarly we have $\tan(\delta_2) = \omega c_2 r_2$. From equation (1) we have
\[
c_2 r_2 - c_1 r_1 = c_1 R_1 - c_2 R_2
\]
on multiplying $\omega$ both sides we have
\[
\omega c_2 r_2 - \omega c_1 r_1 = \omega (c_1 R_1 - c_2 R_2)
\]
But \[
\frac{c_1}{c_2} = \frac{r_4}{r_3}
\]
Therefore the final expression for the dissipation factor is written as
\[
tan(\delta_1) - tan(\delta_2) = \omega c_2 \left( \frac{r_4}{r_3} R_2 - R_2 \right)
\]
Hence if dissipation factor for one capacitor is known. However this method is gives quite inaccurate results for dissipation factor.

**Measurement of Capacitance using Schering Bridge**
This bridge is used to measure to the capacitance of the capacitor, dissipation factor and measurement of relative permittivity. Let us consider the circuit of **Schering bridge** as shown below:
Here, $c_1$ is the unknown capacitance whose value is to be determined with series electrical resistance $r_1$.

c_2$ is a standard capacitor. $c_4$ is a variable capacitor. $r_3$ is a pure resistor (i.e. non inductive in nature). And $r_4$ is a variable non inductive resistor connected in parallel with variable capacitor $c_4$. Now the supply is given to the bridge between the points a and c. The detector is connected between b and d. From the theory of ac bridges we have at balance condition,

$$l_1 \frac{1}{N^2 A}$$

Substituting the values of $z_1$, $z_2$, $z_3$ and $z_4$ in the above equation, we get

$$\left( r_1 + \frac{1}{j \omega c_1} \right) \left( \frac{r_4}{1 + j \omega c_4 r_4} \right) = \frac{r_3}{j \omega c_2}$$

$$(r_1 + \frac{1}{j \omega c_1})r_4 = \frac{r_3}{j \omega c_2} (1 + j \omega c_4 r_4)$$

$$r_1 r_4 - \frac{j r_4}{\omega c_1} = -\frac{j r_3}{\omega c_2} + \frac{r_3 r_4 c_4}{c_2}$$

Equating the real and imaginary parts and the separating we get

$$r_1 = \frac{r_3 c_4}{c_2}$$

$$c_1 = \frac{c_2 r_4}{r_3}$$
Let us consider the phasor diagram of the above Shering bridge circuit and mark the voltage drops across ab, bc, cd and ad as $e_1$, $e_3$, $e_4$ and $e_2$ respectively. From the above Schering bridge phasor diagram, we can calculate the value of $\tan \delta$ which is also called the dissipation factor.

$$\tan \delta = \omega c_1 r_1 = \omega \frac{c_2 r_4}{r^3} \times \frac{r_3 c_4}{c_2} = \omega c_4 r_4$$
Dynamometer type wattmeter works on very simple principle and this principle can be stated as when any current carrying conductor is placed inside a magnetic field, it experiences a mechanical force and due to this mechanical force deflection of conductor takes place.

**Construction and Working Principle of Electrodynamometer Type Wattmeter**

Now let us look at constructional details of electrodynamometer. It consists of following parts.

**Moving Coil**

Moving coil moves the pointer with the help of spring control instrument. Limited of current flows through the moving coil so as to avoid heating. So in order to limit the current we have connected the high value resistor in series with the moving coil. The moving is air cored and is mounted on a pivoted spindle and can move freely. In **electrodynamometer type wattmeter**, moving coil works as pressure coil. Hence moving coil is connected across the voltage and thus the current flowing through this coil is always proportional to the voltage

**Fixed Coil**
The fixed coil is divided into two equal parts and these are connected in series with the load, therefore the load current will flow through these coils. Now the reason is very obvious of using two fixed coils instead of one, so that it can be constructed to carry considerable amount of electric current. These coils are called the current coils of **electrodynamometer type wattmeter**. Earlier these fixed coils are designed to carry the current of about 100 amperes but now the modern wattmeter are designed to carry current of about 20 amperes in order to save power.

**Control System** Out of two controlling systems i.e.
1. Gravity control
2. Spring control, only spring controlled systems are used in these types of wattmeter. Gravity controlled system cannot be employed because they will be appreciable amount of errors.

**Damping System** Air friction damping is used, as eddy current damping will distort the weak operating magnetic field and thus it may leads to error. **Scale** There is uniform scale which is used in these types of instrument as moving coil moves linearly over a range of 40 degrees to 50 degrees on either side. Now let us derive the expressions for the controlling torque and deflecting torques. In order to derive these expressions let us consider the circuit diagram given below:

![Circuit Diagram](image)

We know that instantaneous torque in electrodynamic type instruments is directly proportional to product of instantaneous values of currents flowing through both the coils and the rate of change of flux linked with the circuit. Let $I_1$ and $I_2$ be the instantaneous values of currents in pressure and current coils respectively. So the expression for the torque can be written as:

$$T = I_1 \times I_2 \times \frac{dM}{dx}$$
Where, x is the angle. Now let the applied value of voltage across the pressure coil be

\[ v = \sqrt{2}V\sin\omega t \]

Assuming the electrical resistance to the pressure coil be very high hence we can neglect reactance with respect to its resistance. In this the impedance is equal to its electrical resistance therefore it is purely resistive. The expression for instantaneous current can be written as

\[ I_2 = \frac{\sqrt{2} \times V\sin\omega t}{R_p} \]

If there is phase difference between voltage and electric current, then expression for instantaneous current through current coil can be written as

\[ I_1 = I(t) = \sqrt{2}I\sin(\omega t - \phi) \]

As current through the pressure coil is very very small compared to the current through current coil hence current through the current coil can be considered as equal to total load current. Hence the instantaneous value of torque can be written as

\[ \sqrt{2} \times \frac{V\sin\omega t}{R_p} \times \sqrt{2} \times I \times \sin(\omega t - \phi) \times \frac{dM}{dx} \]

Average value of deflecting torque can be obtained by integrating the instantaneous torque from limit 0 to T, where T is the time period of the cycle.

\[ T_d = deflecting \ torque = \frac{VI}{R_p} \cos\phi \times \frac{dM}{dx} \]

Controlling torque is given by \( T_c = Kx \) where K is spring constant and x is final steady state value of deflection.

**Advantages of Electrodynamometer Type Wattmeter**

Following are the advantages of electrodynamometer type wattmeter and they are written as follows:

1. Scale is uniform upto a certain limit.
2. They can be used for both to measure ac as well dc quantities as scale is calibrated for both.

**Errors in Electrodynamometer Type Wattmeter**

Following are the errors in the electrodynamometer type watt meters:

1. Errors in the pressure coil inductance.
2. Errors may be due to pressure coil capacitance.
3. Errors may be due to mutual inductance effects.
4. Errors may be due connections. (i.e. pressure coil is connected after current coil)
5. Error due to Eddy currents.
6. Errors caused by vibration of moving system.
7. Temperature error.
8. Errors due to stray magnetic field.

**What is Low Power Factor Wattmeter?**

As the name suggests the low power factor meter are the instruments that measures lower values of power factor accurately.

There are two main reasons that would suggests us that we should not use ordinary wattmeter in measuring the low value of power factor.

1. The value of deflecting torque is very low even though we fully excite the current and pressure coils.
2. Errors due pressure coil inductance.

Some modification or adding some new features we can use modified electrodynamic wattmeter or low power factor to measure the low power factor accurately.

(1) The electrical resistance of the ordinary wattmeter’s pressure coil is reduced to low value such that current in the pressure coil circuit is increased, thus it leads to. In this category two cases diagrams arises and these are shown below:

Both the ends of the pressure coil is connected to supply side (i.e. current coil is in series with the load). The supply voltage is equal to the **voltage** across the pressure coil. Thus in this case we
have power shown by the first wattmeter is equal to the power loss in the load plus power loss in the current coil.

Mathematically \( P_1 = \text{power consumed by load} + I^2 R_1 \)

In the second category, the current coil is not in series with the load and the voltage across the pressure coil is not equal to the applied voltage. The voltage across pressure coil is equal to the voltage across the load. In this power shown by the second watt meter is equal to the power loss in the load plus the power loss in the pressure coil.

Mathematically \( P_2 = \text{power consumed by load} + I^2 R_2 \)

From the above discussion we conclude that in both cases we have some amount of errors hence there is need to do some modification in above circuits to have minimum error.

The modified circuit is shown below: We have used here a special coil called compensating coil, it carries current equal to the sum of two currents i.e load current plus pressure coil current. The pressure coil is placed such that the field produced by the compensating coil is opposed by the field produced by pressure coil as shown in the above circuit diagram.

Thus the net field is due to the current I only. Hence by this way error caused by pressure coil can be neutralised.

(2) We require compensating coil in the circuit in order to make the low power factor meter. It is the second modification that we have discussed in detail above.
(3) Now the third point deals with the compensation of the inductance of pressure coil, which can be achieved by doing modification in above circuit.

Now let us derive an expression for the correction factor for pressure coil inductance. And from this correction factor we are going to derive an expression for error due to inductance of pressure coil. If we consider the inductance of pressure coil we don't have voltage across pressure in phase with the applied voltage. Hence it that case it lag by an angle

\[ b = \tan^{-1} \frac{W_i}{R + r_p} \]

Where, \( R \) is electrical resistance in series with pressure coil, \( r_p \) is pressure coil resistance, here we also conclude that the current in the current coil is also lagging by some angle with the current in pressure coil. And this angle is given by \( C = A - b \). At this time reading of the voltmeter is given by

\[ \frac{V I \cos(C) \frac{dM}{dx}}{K + R_p} \ldots \ldots \ldots (1) \]

Where, \( R_p \) is \( (r_p+R) \) and \( x \) is angle. If we ignore the effect of inductance of pressure i.e putting \( b = 0 \) we have expression for true power as

\[ \frac{V I \cos(A) \frac{dM}{dx}}{K \cdot R_p} \ldots \ldots \ldots (2) \]
On taking ratio of equations (2) and (1) we have expression for correction factor as written below:

$$\frac{\cos(A)}{\cos(b)\cos(A - b)}$$

And from this correction factor error can be calculated as

$$Error = \{1 - (\text{correction factor})\} \times (\text{actual reading of the voltmeter})$$

On substituting the value of correction factor and taking suitable approximation we have expression for error as $V\sin(A)\tan(b)$.

Now we know that the error caused by pressure coil inductance is given by the expression $e = V\sin(A)\tan(b)$, if power factor is low (i.e. in our case the value of $\phi$ is large hence we have large error). Thus in order to avoid this situation we have connect the variable series resistance with a capacitor as shown in the above figure. This final modified circuit so obtained is called low power factor meter. A modern low power factor meter is designed such that it gives high accuracy while measuring power factors even lower than 0.1.

**TWO-ELEMENT WATTMETER FOR THREE-PHASE SYSTEM**

two single-phase wattmeters were used to measure the power in a three-phase, three-wire system. The two single-phase wattmeters can be combined into a single instrument
The scale of this instrument indicates the sum or difference of the power values indicated by the separate meters. To make the single wattmeter, two sets of potential coils are mounted on a single shaft. Also, two sets of field coils are mounted on the instrument frame so that they have the proper relationship to the armature coils. In this way, each of two power measuring mechanisms develops a torque that is proportional to the power in the circuit to which it is connected. These torque values are added to obtain the total power in the three-phase, three-wire circuit. If the power factor of the system is less than 0.5, the torque of one mechanism opposes that of the second mechanism. The difference between the torque values is the power indication.

A wattmeter containing two dynamometer mechanisms is called a two-element wattmeter.

Power Factor Meters

Now there are two types of power factor meters-

1. Electrodynamometer type
2. Moving iron type.

Let us study electrodynamometer type first.

Electrodynamometer Type Power Factor Meter
In **electrodynamometer type power factor meter** there are further two types on the basis of supply voltage

1. Single phase
2. Three phase.

The general circuit diagram of single phase electrodynamometer power factor meter is given below.

![Diagram of single phase electrodynamometer power factor meter]

Now the pressure coil is split into two parts one is purely inductive another is purely resistive as shown in the diagram by resistor and inductor. At present the reference plane is making an angle $A$ with coil 1. And the angle between both the coils 1 and 2 is $90^\circ$. Thus the coil 2 is making an angle $(90^\circ + A)$ with the reference plane. Scale of the meter is properly calibrated as shown the value values of cosine of angle $A$. Let us mark the electrical resistance connected to coil 1 be $R$ and inductor connected to coil 2 be $L$. Now during measurement of power factor the values of $R$ and $L$ are adjusted such that $R = wL$ so that both coils carry equal magnitude of current. Therefore the current passing through the coil 2 is lags by $90^\circ$ with reference to current in coil 1 as coil 2 path is highly inductive in nature. Let us derive an expression for deflecting torque for this power factor meter. Now there are two deflecting torques one is acting on the coil 1 and another is acting on the coil 2. The coil winding are arranged such that the two torques produced, are opposite to each other and therefore pointer will take a position where the two torques are equal. Let us write a mathematical expression for the deflecting torque for coil 1-
\[ T_1 = KVIM \cos A \sin B \]

Where \( M \) is the maximum value of mutual inductance between the two coils, \( B \) is the angular deflection of the plane of reference. Now the mathematical expression for the deflecting torque for coil 2 is:

\[ T_2 = KVIM \cos(90 - A) \sin(90 + B) = KVIM \sin A \cos B \]

At equilibrium we have both the torque as equal thus on equating \( T_1 = T_2 \) we have \( A = B \). From here we can see that the deflection angle is the measure of phase angle of the given circuit. The phasor diagram is also shown for the circuit such that the current in the coil 1 is approximately at an angle of 90° to current in the coil 2.

Given below are some of the advantages and disadvantages of using electrodynamic type power factor meters.

**Advantages of Electrodynamic Type Power Factor Meters**

1. Losses are less because of minimum use of iron parts and also give less error over a small range of frequency as compared to moving iron type instruments.
2. They have a high torque to weight ratio.

**Disadvantages of Electrodynamic Type Power Factor Meters**

1. Working forces are small as compared to moving iron type instruments.
2. The scale is not extended over 360°.
3. Calibration of electrodynamometer type instruments are highly affected by the changing the supply voltage frequency.
4. They are quite costly as compared to other instruments.
**WATT HOUR METER**

**Watt-hour meter** is in fact a measuring device which can evaluate and records the electrical power passing through a circuit in a certain time. By implementing the Watt-hour meter, we can know how much amount of electrical energy is used by a consumer or a residence or an electrically powered device or a business. Electrical utilities install these meters at their consumer’s place to evaluate the electrical usage for the purpose of billing. The reading is taken in each one billing period. Usually, the billing unit is Kilowatt-hour (kWh). This is equal to the total usage of electrical energy by a consumer of one kilowatt during a period of one hour and it is also equal to 3600000 joules. The Watt-Hour Meter is often referred as energy meter or electric meter or electricity meter or electrical meter.

Mainly the watt-hour meter comprises of a tiny motor and a counter. The motor will operate by diverting exact fraction of **current** which is flowing in the circuit to be measured.

**Electromechanical Type Induction Meter**

In this type of meter, a non-magnetic and electrically conductive aluminium metal disc is made to revolve in a **magnetic field**. The rotation is made possible with the power passing through it. The rotation speed is proportional to the power flow through the meter. Gear trains and counter mechanisms are incorporated to integrate this power. This meter works by counting the total number of revolutions and it is relative to the usage of energy.

A series magnet is connected in series with the line and that comprises of a coil of few turns with thick wire. A shunt magnet is connected in shunt with the supply and comprises of a coil of large number of turns with thin wire. A braking magnet which is a permanent magnet is included for stopping the disc at the time of power failure and to place the disc in position. This is done by applying a force opposite to the rotation of the disc.
A flux is produced by the series magnet that is directly proportional to the current flow and another flux is produced by the shunt magnet corresponding to the voltage. Because of the inductive nature, these two fluxes lag each other by 90°. An eddy current is developed in the disc which is the interface of the two fields. This current is produced by a force that is corresponding to the product of instantaneous current, voltage and the phase angle among them. A break torque is developed on the disc by the braking magnet positioned over one side of the disc. The speed of the disc becomes constant when the following condition is achieved, Braking torque = Driving torque. The gear arrangement linked with the shaft of the disc is implemented for recording the number of revolution. This is for single phase AC measurement. Additional number of coils can be implemented for different phase configuration.

\[
Power = \frac{3600.Kh}{\text{time in sec for one revolution of the disc}}
\]

**INDUCTION TYPE METERS**

The principle of working and construction of induction type meter is very simple and easy to understand that's why these are widely used in measuring energy in domestic as well as industrial world. In all induction meters we have two fluxes which are produced by two different alternating currents on a metallic disc. Due to alternating fluxes there is an induced emf, the emf
produced at one point (as shown in the figure given below) interacts with the alternating current of the other side resulting in the production of torque

Similarly, the emf produced at the point two interacts with the alternating current at point one, resulting in the production of torque again but in opposite direction. Hence due to these two torques which are in different directions, the metallic disc moves. This is basic principle of working of an induction type meters. Now let us derive the mathematical expression for deflecting torque. Let us take flux produced at point one be equal to $F_1$ and the flux and at point two be equal to $F_2$. Now the instantaneous values of these two flux can written as:

$$F_1 = F_{m1} \sin \omega t, \quad F_2 = F_{m2} \sin(\omega t - B)$$

Where, $F_{m1}$ and $F_{m2}$ are respectively the maximum values of fluxes $F_1$ and $F_2$, $B$ is phase difference between two fluxes. We can also write the expression for induced emf's at point one be

$$E_1 = -\frac{d(F_1)}{dt} \quad and \quad E_2 = -\frac{d(F_2)}{dt}$$

at point two. Thus we have the expression for eddy currents at point one is

$$E_1 = -\frac{d(F_1)}{dt} \quad and \quad E_2 = -\frac{d(F_2)}{dt}$$
\[ I_1 = \frac{E_1}{Z} = K \times f \times F_1 \]

Where, \( K \) is some constant and \( f \) is frequency. Let us draw phasor diagram clearly showing \( F_1 \), \( F_2 \), \( E_1 \), \( E_2 \), \( I_1 \) and \( I_2 \). From phasor diagram, it clear that \( I_1 \) and \( I_2 \) are respectively lagging behind \( E_1 \) and \( E_2 \) by angle \( A \).

The angle between \( F_1 \) and \( F_2 \) is \( B \). From the phasor diagram the angle between \( F_2 \) and \( I_1 \) is \((90 - B + A)\) and the angle between \( F_1 \) and \( I_2 \) is \((90 + B + A)\). Thus we write the expression for deflecting torque as

\[ T_{d1} = K \times F_2 \times I_1 \times \cos(90 - B + A) = K \times F_1 \times F_2 \times \frac{f}{Z} \cos(90 - B + A). \]

Similarly the expression for \( T_{d2} \) is

\[ T_{d2} = K \times F_1 \times F_2 \times \frac{f}{Z} \cos(90 + A + B). \]
The total torque is $T_{d1} - T_{d2}$, on substituting the the value of $T_{d1}$ and $T_{d2}$ and simplying the expression we get

$$T_{d1} - T_{d2} = K \times F_1 \times F_2 \times \frac{f}{Z} \sin(B)\cos(A).$$

Which is known as the general expression for the deflecting torque in the **induction type meters**. Now there are two types of induction meters and they are written as follows:

- Single phase type
- Three phase type induction meters.

Here we are going to discuss about the single phase induction type in detail. Given below is the picture of single phase induction type meter.
Single phase induction type energy meter consists of four important systems which are written as follows:

Driving System: Driving system consists of two electromagnets on which pressure coil and current coils are wounded, as shown above in the diagram. The coil which consisted of load current is called current coil while coil which is in parallel with the supply voltage (i.e. voltage across the coil is same as the supply voltage) is called pressure coil. Shading bands are wounded on as shown above in the diagram so as to make angle between the flux and applied voltage equal to 90 degrees. Moving System: In order to reduce friction to greater extent floating shaft energy meter is used, the friction is reduced to greater extinct because the rotating disc which is made up of very light material like aluminium is not in contact with any of the surface. It floats in the air. One question must be arise in our mind is that how the aluminium disc float in the air? To answer this question we need to see the constructional details of this special disc, actually it consists of small magnets on both upper and lower surfaces. The upper magnet is attracted to an electromagnet in upper bearing while the lower surface magnet also attracts towards the lower bearing magnet, hence due to these opposite forces the light rotating aluminium disc floats.

Braking System: A permanent magnet is used to produce breaking torque in single phase induction energy meters which are positioned near the corner of the aluminium disc.

Counting System: Numbers marked on the meter are proportion to the revolutions made by the aluminium disc, the main function of this system is to record the number of revolutions made by the aluminium disc. Now let us look at the working operation of the single phase induction meter. In order to understand the working of this meter let us consider the diagram given below:
Here we have assumed that the pressure coil is highly inductive in nature and consists of very large number of turns. The current flowing in the pressure coil is $I_p$ which lags behind voltage by an angle of 90 degrees. This current produces flux $F$. $F$ is divided into two parts $F_g$ and $F_p$.

1. $F_g$ which moves on the small reluctance part across the side gaps.
2. $F_p$: It is responsible for the production of driving torque in the aluminium disc. It moves from high reluctance path and is in phase with the current in the pressure coil. $F_p$ is alternating in nature and thus emf $E_p$ and current $I_p$. The load current which is shown in the above diagram is flowing through the current coil produces flux in the aluminium disc, and due this alternating flux there on the metallic disc, an eddy current is produced which interacts with the flux $F_p$ which results in production of torque. As we have two poles, thus two torques are produced which are opposite to each other. Hence from the theory of induction meter that we have discussed already above the net torque is the difference of the two torques.

**Advantages of Induction Type Meters**

Following are the advantages of induction type meters:

1. They are inexpensive as compared to moving iron type instruments.
2. They have high torque is to weight ratio as compared to other instruments.
3. They retain their accuracy over wide range of temperature as well as loads.
THREE PHASE ENERGY METER

Introduction

It is well established that for measurement of total power or energy in a n-conductor system, it is required to use a meter with (n-1) elements. The principle of single phase energy meter can as well be extended to obtain a poly-phase energy meter, in particular a three phase energy meter. Usually, a three phase energy meter is available as a 2-element meter or 3-element meter, each element being similar in construction to the single phase meter and all elements mounted on a common shaft. The torque developed by each element is summed up mechanically and the total number of revolutions made by the shaft is proportional to the total three phase energy consumption.

Construction, Operation and Testing

In a two-element, three phase energy meter the two discs are mounted on a common spindle and each disc has its own brake magnet. The moving system drives a single gear train. Each unit is provided with its own copper shading ring, shading band, friction compensator, etc., for adjustments to be made to obtain the correct reading.

![Diagram of Three Phase Energy Meter](image)

Figure 8.7 Three Phase Energy Meter
A two element energy meter used for three phase energy measurements in three phase three wire systems, is schematically shown in figure 8.7. It is needful that for the same power/energy, the driving torque should be equal in the two elements. This is checked by torque adjustment. For torque adjustment, the two current coils are connected in series opposition and the two potential coils are connected in parallel. Full load current is allowed to pass through the current coil. This set up causes the two torques to be in opposition and so, if the torques are equal, then the disc should not move. If there is any slight motion indicating inequality of the two torques, then the magnetic shunt is adjusted until the disc stalls. Thus the torque balancing is obtained before testing the meter. The friction compensator and brake magnet positions are adjusted to each of the two/three elements separately, treating each of them as a single phase element on single phase AC supply. The calibration of three phase meter can also be performed in a similar manner, as that described earlier, for single phase energy meters.

**ERRORS IN ENERGY METERS**

The energy measurements by energy meters involve errors owing to many sources and reasons as follows:

1. **Errors in driving system** include errors due to incorrect magnitude of flux values, phase angles, etc. and lack of symmetry in magnetic circuit.

2. **Errors in braking system** such as changes in the strength of brake magnet, changes in disc resistance, self braking effect of series magnet flux and abnormal friction of the moving parts.

3. Errors in registering system are also expected to be present since they involve mechanical parts. They are taken care of by calibration of the meter.

4. **Errors caused due to friction, overloads, phase angle** variations, temperature effects, creeping of the meter, etc. These errors are avoided by correct adjustments made using the various compensator facility provided on the meter.

**Adjustments**

**Full Load UPF Adjustment**: The potential coil is connected across rated supply voltage and rated full load current at unity power factor is passed through the current coil. The brake magnet position is adjusted to vary the braking torque so that the moving system moves at correct speed.
Lag or LPF adjustment: It is clear from equation (8.10) that the energy meter will register correct value only if the angle between the shunt magnet flux, $f_P$ and the supply voltage, $V$ is 90° ($D = 90°$). Hence the pressure coil should be designed to be highly inductive. Also, various lag adjustment devices are made use of for this purpose. For LPF adjustments, the pressure coil is connected across the rated supply voltage and rated full load current at 0.5 lagging power factor is passed through the current coil. The lag device is adjusted until the meter runs at true speed.

Light Load UPF Adjustment: Firstly, full load UPF and LPF adjustments are made on the meter until it runs at correct speed. Then rated supply voltage is applied across the pressure coil and a very low current of 5-10% of full load value is passed through the meter at unity power factor. The light load adjustment is done so that the meter runs at proper speed.

Creep Adjustment: Firstly, full load UPF and light load adjustments are made for correct speeds at both the loads and the performance is rechecked at 0.5 power factor. Then, as a final check on all the above adjustments, the pressure coil is excited by 110% of the rated voltage with zero load current. If the light load adjustment is proper, the meter should not creep under these conditions. If the error still persists, then all the above adjustments are carried out once again.

UNIT 4

INSTRUMENT TRANSFORMERS

Introduction of Instrument Transformers

Instrument Transformers are used in AC system for measurement of electrical quantities i.e. voltage, current, power, energy, power factor, frequency. Instrument transformers are also used with protective relays for protection of power system. Basic function of Instrument transformers is to step down the AC System voltage and current. The voltage and current level of power system is very high. It is very difficult and costly to design the measuring instruments for measurement of such high level voltage and current. Generally measuring instruments are designed for 5 A and 110 V.
The measurement of such very large electrical quantities, can be made possible by using the Instrument transformers

Advantages of Instrument Transformers
1. The large voltage and current of AC Power system can be measured by using small rating measuring instrument i.e. 5 A, 110 – 120 V.
2. By using the instrument transformers, measuring instruments can be standardized. Which results in reduction of cost of measuring instruments. More ever the damaged measuring instruments can be replaced easy with healthy standardized measuring instruments.
3. Instrument transformers provide electrical isolation between high voltage power circuit and measuring instruments. Which reduces the electrical insulation requirement for measuring instruments and protective circuits and also assures the safety of operators.
4. Several measuring instruments can be connected through a single transformer to power system.
5. Due to low voltage and current level in measuring and protective circuit, there is low power consumption in measuring and protective circuits.

Types of Instrument Transformers
Instrument transformers are of two types –
1. Current Transformer (C.T.)
2. Potential Transformer (P.T.)

Current Transformer (C.T.)
Current transformer is used to step down the current of power system to a lower level to make it feasible to be measured by small rating Ammeter (i.e. 5A ammeter). A typical connection diagram of a current transformer is shown in figure below.

Primary of C.T. is having very few turns. Sometimes bar primary is also used. Primary is connected in series with the power circuit. Therefore, sometimes it also called series transformer.
The secondary is having large no. of turns. Secondary is connected directly to an ammeter. As the ammeter is having very small resistance. Hence, the secondary of current transformer operates almost in short circuited condition. One terminal of secondary is earthed to avoid the large voltage on secondary with respect to earth. Which in turns reduce the chances of insulation breakdown and also protect the operator against high voltage. More ever before disconnecting the ammeter, secondary is short circuited through a switch ‘S’ as shown in figure above to avoid the high voltage build up across the secondary.

### Potential Transformer (P.T.)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Connected in series with power circuit.</td>
<td>Connected in Parallel with Power circuit.</td>
</tr>
<tr>
<td>2</td>
<td>Secondary is connected to Ammeter.</td>
<td>Secondary is connected to Voltmeter.</td>
</tr>
<tr>
<td>3</td>
<td>Secondary works almost in short circuited condition.</td>
<td>Secondary works almost in open circuited condition.</td>
</tr>
<tr>
<td>4</td>
<td>Primary current depends on power circuit current.</td>
<td>Primary current depends on secondary burden.</td>
</tr>
<tr>
<td>5</td>
<td>Primary current and excitation vary over wide range with change of power circuit current</td>
<td>Primary current and excitation variation are restricted to a small range.</td>
</tr>
<tr>
<td>6</td>
<td>One terminal of secondary is earthed to avoid the insulation break down.</td>
<td>One terminal of secondary can be earthed for Safety.</td>
</tr>
<tr>
<td>7</td>
<td>Secondary is never be open circuited.</td>
<td>Secondary can be used in open circuit condition.</td>
</tr>
</tbody>
</table>

**Error in PT or Potential Transformer or VT or Voltage Transformer**
I_s - Secondary current.

E_s - Secondary induced emf.

V_s - Secondary terminal voltage.

R_s - Secondary winding resistance.

X_s - Secondary winding reactance.

I_p - Primary current.

E_p - Primary induced emf.

V_p - Primary terminal voltage.

R_p - Primary winding resistance.

X_p - Primary winding reactance.

K_T - Turns ratio = Numbers of primary turns/number of secondary turns.

I_0 - Excitation current.

I_m - Magnetizing component of I_0.

I_w - Core loss component of I_0.

Φ_m - Main flux.

β - Phase angle error.

As in the case of current transformer and other purpose electrical power transformer, total primary current I_p is the vector sum of excitation current and the current equal to reversal of secondary current multiplied by the ratio 1/K_T.
Hence, \( I_p = \frac{I_0 + I_s}{K_T} \)

If \( V_p \) is the system voltage applied to the primary of the PT, then voltage drops due to resistance and reactance of primary winding due to primary current \( I_p \) will come into picture. After subtracting this voltage drop from \( V_p \), \( E_p \) will appear across the primary terminals. This \( E_p \) is equal to primary induced emf. This primary emf will transform to the secondary winding by mutual induction and transformed emf is \( V_s \). Again this \( E_s \) will be dropped by secondary winding resistance and reactance, and resultant will actually appear across the burden terminals and it is denoted as \( V_s \). So, if system voltage is \( V_p \), ideally \( V_p/K_T \) should be the secondary voltage of PT, but in reality; actual secondary voltage of PT is \( V_s \).

**Voltage Error or Ratio Error in Potential Transformer (PT) or Voltage Transformer (VT)**

The difference between the ideal value \( V_p/K_T \) and actual value \( V_s \) is the voltage error or ratio error in a potential transformer, it can be expressed as,

\[
\text{% voltage error} = \frac{V_p - K_T V_s}{V_p} \times 100 \%
\]

**Phase Error or Phase Angle Error in Potential or Voltage Transformer**

The angle 'β' between the primary system voltage \( V_p \) and the reversed secondary voltage vectors \( K_T V_s \) is the phase error.

**Cause of Error in Potential Transformer**

The voltage applied to the primary of the potential transformer first drops due to the internal impedance of the primary. Then it appears across the primary winding and then transformed proportionally to its turns ratio, to the secondary winding. This transformed voltage across the secondary winding will again drop due to the internal impedance of the secondary, before appearing across burden terminals. This is the reason of errors in potential transformer.

**Ratio Error Phase Angle Error in Current Transformer**

**Definition of Instrument Transformer**

Instrument transformers means current transformer and voltage transformer are used in electrical power system for stepping down currents and voltages of the system for metering and protection purpose.

**Current Transformer(CT)**
A CT is an instrument transformer in which the secondary current is substantially proportional to primary current and differs in phase from it by ideally zero degree.

**Error in Current Transformer or CT**
But in an actual CT, errors with which we are connected can best be considered through a study of phasor diagram for a CT,

- $I_s$ - Secondary current
- $E_s$ - Secondary induced emf.
- $I_p$ - Primary current.
- $E_p$ - Primary induced emf.
- $K_T$ - Turns ratio = Numbers of secondary turns/number of primary turns.
- $I_0$ - Excitation current.
- $I_m$ - Magnetizing component of $I_0$.
- $I_w$ - Core loss component of $I_0$.
- $\Phi_m$ - Main flux. Let us take flux as reference.

EMF $E_s$ and $E_p$ lags behind the flux by 90°. The magnitude of the passers $E_s$ and $E_p$ are proportional to secondary and primary turns. The excitation current $I_0$ which is made up of two components $I_m$ and $I_w$. The secondary current $I_0$ lags behind the secondary induced emf $E_s$ by an angle $\Phi_s$. The secondary current is now transferred to the primary side by reversing $I_s$ and multiplied by the turns ratio $K_T$. The total current flows through the primary $I_p$ is then vector sum of $K_T I_s$ and $I_0$. 
The Current Error or Ratio Error in Current Transformer or CT
From above passer diagram it is clear that primary current $I_p$ is not exactly equal to the secondary current multiplied by turns ratio, i.e. $K_T I_s$. This difference is due to the primary current is contributed by the core excitation current. The error in current transformer introduced due to this difference is called current error of CT or sometimes ratio error in current transformer.

Hence, the percentage current error = $\frac{|I_p| - |K_T I_s|}{I_p} \times 100 \%$

Phase Error or Phase Angle Error in Current Transformer

For an ideal CT the angle between the primary and reversed secondary current vector is zero. But for an actual CT there is always a difference in phase between two due to the fact that primary current has to supply the component of the exiting current. The angle between the above two phases is termed as phase angle error in current transformer or CT. Here in the pharos diagram it is $\beta$ the phase angle error is usually expressed in minutes.

Cause of Error in Current Transformer
The total primary current is not actually transformed in CT. One part of the primary current is consumed for core excitation and remaining is actually transformed with turns ratio of CT so there is error in current transformer means there are both ratio error in current transformer as well as a phase angle error in current transformer.

How to Reduce Error in Current Transformer
It is desirable to reduce these errors, for better performance. For achieving minimum error in current transformer, one can follow the following,

1. Using a core of high permeability and low hysteresis loss magnetic materials.
2. Keeping the rated burden to the nearer value of the actual burden.
3. Ensuring minimum length of flux path and increasing cross-sectional area of the core, minimizing joint of the core.
4. Lowering the secondary internal impedance.

Working Principle of Potentiometer
This is a very basic instrument used for comparing emf two cells and for calibrating ammeter, voltmeter and watt-meter. The basic working principle of potentiometer is very very simple. Suppose we have connected two battery in head to head and tale to tale through a galvanometer. That means the positive terminals of both battery are connected together and negative terminals are also connected together through a galvanometer as shown in the figure below.
it is clear that if the **voltage** of both battery cells is exactly equal, there will be no circulating **current** in the circuit and hence the galvanometer shows null deflection. The **working principle of potentiometer** depends upon this phenomenon.

Now let's think about another circuit, where a battery is connected across a resistor via a switch and a rheostat as shown in the figure below voltage-drop-calculation/ across the resistor. As there is a voltage drop across the resistor, this portion of the circuit can be considered as a voltage source for other external circuits. That means anything connected across the resistor will get voltage. If the resistor has uniform cross section throughout its length, the electrical resistance per unit length of the resistor is also uniform throughout its length.

voltage drop per unit length of the resistor is also uniform. Suppose the current through the resistor is i A and resistance per unit length of the resistor is \( r \ \Omega \). Then the voltage appears per unit length across the resistor would be ‘ir’ ans say it is v volt.

positive terminal of a standard cell is connected to point A on the sliding resistor and negative terminal of the same is connected with a galvanometer. Other end of the galvanometer is in contact with the resistor via a sliding contact as shown in the figure above. By adjusting this sliding end, a point like B is found where, there is no current through the galvanometer, hence no deflection of galvanometer.

That means emf of the standard cell is just balanced by the voltage-drop-calculation/ appears across AB. Now if the distance between point A and B is L, then it can be written emf of standard cell \( E = Lv \) volt. As v (voltage drop per unit length of the sliding resistor) is known and L is measured from the scale attached to the resistor, the value of E i.e. emf of standard cell can also be calculated from the above simple equation very easily.
DC POTENTIOMETER CAN COMPARE EMFS OF TWO DIFFERENT CELLS

Two cells whose emf's are to be compared are joined as shown in the figure below. The positive terminals of the cells and source battery are joined together. The negative terminals of the cells are joined with the galvanometer in turn through a two way switch. The other end of the galvanometer is connected to a sliding contact on the resistor. Now by adjusting sliding contact on the resistor, it is found that the null deflection of galvanometer comes for first cell at a length of $L$ on the scale and after positioning to way switch to second cell and then by adjusting the sliding contact, it is found that the null deflection of galvanometer comes for that cell at a length of $L_1$ on the scale. Let's think of the first cell as standard cell and it's emf is $E$ and second cell is unknown cell whose emf is $E_1$. Now as per above explanation, $E = Lv$ volt and $L_1 = L_1v$ volt Dividing one equation by other, we get

$$\frac{E_1}{E} = \frac{L_1}{L}$$

As the emf of the standard cell is known, hence emf of the unknown cell can easily be determined.
AC Potentiometer

The Potentiometer is an instrument which measures unknown voltage by balancing it with a known voltage. The known source may be DC or AC. The working phenomenon of DC potentiometer and AC potentiometer is same. But there is one major difference between their measurements, DC potentiometer only measures the magnitude of the unknown voltage. Whereas, AC potentiometer measures both the magnitude and phase of unknown voltage by comparing it with known reference. There are two types of AC potentiometers:

1. Polar type potentiometer.
2. Coordinate type potentiometer.

Polar type Potentiometer

In such type of instruments, two separate scales are used to measure magnitude and phase angle on some reference of the unknown e.m.f. There is a provision on the scale that it could read phase angle up to 3600. It has electrodynamometer type ammeter along with DC potentiometer and phase-shifting transformer which is operated by single phase supply. In phase-shifting transformer, there is a combination of two ring-shaped laminated steel stators connected perpendicularly to each other as shown in the figure. One is directly connected to power supply and the other one is connected in series with variable resistance and capacitor. The function of the series components is to maintain constant AC supply in the potentiometer by doing small adjustments in it.
Between the stators, there is laminated rotor having slots and winding which supplies voltage to the slide-wire circuit of the Potentiometer. When current starts flowing from stators, the rotating field is developed around the rotor and due to it e.m.f. is induced in the rotor winding. The phase displacement of the rotor emf is equal to rotor movement angle from its original position and it is related to stator supply voltage. The whole arrangement of winding are done in such a way that the magnitude of the induced emf in the rotor may change but it does not affect the phase angle and it can be read on the scale fixed on the top of the instrument.

The induced emf in rotor winding by stator winding 1 can be expressed as

\[ E_1 = K I \sin\omega t \cos \phi \quad \ldots \ldots \ldots (1) \]

The induced emf in the rotor winding by the stator winding 2,

\[ E_2 = K I \sin(\omega t + 90^\circ) \cos(\phi + 90^\circ) \]
\[ = -K I \cos \omega t \sin \phi \quad \ldots \ldots \ldots (2) \]

From equation (1) and (2), we get

\[ E = K I (\sin \omega t \cos \phi - \cos \omega t \sin \phi) \]

Therefore, resultant induced emf in the rotor winding due to two stator winding

\[ E = K I \sin (\omega t - \phi) \]

Where, \( \phi \) gives the phase angle.
COORDINATE TYPE POTENTIOMETER

In coordinate AC potentiometer, two separate potentiometers are caged in one circuit as shown in the figure. The first one is named as the in-phase potentiometer which is used to measure the in-phase factor of an unknown e.m.f. and the other one is named as quadrature potentiometer which measures quadrature part of the unknown e.m.f. the sliding contact AA’ in the in-phase potentiometer and BB’ in quadrature potentiometer are used for obtaining the desired current in the circuit. By adjusting rheostat R and R’ and sliding contacts, the current in the quadrature potentiometer becomes equal to the current in the in-phase potentiometer and a variable galvanometer shows the null value. S₁ and S₂ are signs changing switches which are used to change the polarity of the test voltage if it is required for balancing the Potentiometer. There are two step-down transformers T₁ and T₂ which isolate potentiometer from the line and give an earthed screens protection between the winding. It also supplies 6 volts to potentiometers. Now to measure unknown e.m.f. its terminals are connected across sliding contacts AA’ using selector switch S₃. By doing some adjustments in sliding contacts and rheostat, the whole circuit gets balanced and galvanometer reads zero at the balanced condition. Now the in-phase component \( V_A \) of the unknown e.m.f. is obtained from the in-phase potentiometer and quadrature component \( V_B \) is obtained from quadrature potentiometer.

Coordinate AC potentiometer
Thus, the resultant voltage of the coordinate AC potentiometer is

\[ V = (V_A^2 + V_B^2)^{1/2} \]

And the phase angle is given by

\[ \varphi = \tan^{-1}(V_B/V_A) \]

**Applications of AC Potentiometer**

2. Calibration of voltmeter.